



JUNE 13, 2003

Math 226 Midterm # 2

Fully answer all the questions in the booklets provided.   
No communication of any kind with your fellow students.   
single  $8\frac{1}{2} \times 11$  hand written sheet. Time: 2 hours.

Question 1. Determine the radii of convergence of each of the following power series.

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- (a)  $\sum_{n=1}^{\infty} x^n \sin\left(\frac{1}{2^n}\right)$       (b)  $\sum_{n=1}^{\infty} n!(x-5)^n$   
(c)  $\sum_{n=1}^{\infty} \frac{x^n}{(n!)^3}$       (d)  $\sum_{n=1}^{\infty} \frac{x^n}{3^n \sqrt{n}}$   
(e)  $\sum_{n=1}^{\infty} \frac{x^{2n}}{4^n n^2}$

Question 2. Determine the interval of convergence of each of the following power series.

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- (a)  $\sum_{n=1}^{\infty} \sqrt{n} x^n$       (b)  $\sum_{n=1}^{\infty} \frac{x^{2n}}{n^2}$       (c)  $\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{2^n (n+3)}$

Question 3. Suppose  $f(x)$  is a function which has Taylor series

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$$f(x) = 2x - \frac{1}{2}x^2 + 5x^3 + \dots$$

Calculate, up to and including terms of order  $x^3$ , the Taylor expansions of the functions

- (a)  $f(x) \sin x$       (b)  $f(x) + 2\sqrt{1-x}$       (c)  $xf'(x)$ .

Question 4. Find the Taylor series, up to and including terms of order  $x^2$ , of the function

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$$y = \ln(1 + 2 \sin x).$$

Question 5. Find the general solution to the following differential equations.

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- (a)  $\frac{dy}{dx} = \frac{y(x^2 - 4)}{x}$       (b)  $\frac{1}{t} \frac{dy}{dt} - y + 2 = 0.$

Question 6. Find the general solution to the following differential equations.

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- (a)  $\frac{dy}{dx} + \frac{4x}{1+x^2}y = \frac{4x}{1+x^2}$       (b)  $x \frac{dy}{dx} + (1+x)y - 1 = 0.$

Question 7. Solve the following initial value problems.

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- (a)  $\frac{dy}{dx} + y = x^2, \quad y(1) = 2$       (b)  $\frac{dy}{dt} = \frac{y-1}{t-1}, \quad y(3) = 2.$